Review Sheet for Midterm I

Math 20: Introduction to Linear Algebra and Multivariable Calculus

October 21, 2004

Review Materials and Problems

Start by making sure you know how to do all of your homework problems. Solutions are posted on the course web site. You can also try additional, unassigned problems from any section you don't understand completely

There is a set of review problems after each Chapter in Lay. The answers to the odd problems are in the back of the book.

- For Chapter 1, don't worry about review problems 20–24
- For Chapter 2, skip problems 11, 12, 19, 20

For topics not covered in the book, check the course web site for more information. Google is a seemingly endless source of references, including many other linear algebra course websites.

Review Topics

These are things you should know how to do in preparing for the first exam, organized by topic.

Systems of Linear Equations (Section 1.1 of Lay)

- Convert a system of linear equations into a matrix and vice versa
- Use row operations to solve a system of linear equations or determine its inconsistency

Row Reduction and Echelon Forms (1.2)

- Determine whether a matrix is in row echelon form
- Determine whether a matrix is in reduced row echelon form
- Use row operations (Gaussian Elimination) to reduce a matrix to REF or RREF
- Read off solutions to a system of linear equations from the RREF of its augmented matrix
- Give the parametric description of solutions to SLEs
- Identify free and basic variables are and use them to gain information about the solution set to a SLE

Vector Equations (1.3)

- Use the geometric properties of vectors
- Use the algebraic properties of addition and scalar multiplication of vectors
- Demonstrate the concept of linear combination of a set of vectors
- Convert between vector equations and SLEs
- Given a finite set of vectors, determine whether one is in the span of the rest

The Matrix Equation Ax=b (1.4)

- Multiply a matrix by a vector in two different ways
- Convert between vector equations and matrix equations
- Understand the correspondence between solutions to matrix equations, writing a vector as a linear combination of other vectors, stating a vector is in the span of other vectors, and descriptions of the RREF of a matrix (Theorem 1.4)

Solution Sets of Linear Systems (1.5)

- Identify homogeneous and nonhomogeneous equations
- Give the parametric vector form of the solution set to a syste

Applications of Linear Systems (1.6)

- Convert a case ("story") problem into a system of linear equations

Linear Independence (1.7)

- Determine whether a given set of vectors is linearly dependent or independent

Matrix Operations (2.1)

- Add matrices
- Multiply matrices (this includes knowing when the multiplication is defined)
- Use both the entry-by-entry and column-vector definitions of these matrix operations
- Take the transpose of a given matrix
- Use the rules for algebra of matrices (Theorems 2.1–2.3)

The Inverse of a Matrix (2.2)

- Compute the inverse of a 2×2 matrix using the shortcut (Theorem 2.4)

- Determine the invertibility (and compute the inverse) of a $n \times n$ matrix using Gaussian Elimination
- Solve a system of linear equations using the inverse of the coefficient matrix

Characterizations of Invertible Matrices (2.3)

- Use the many, many equivalent conditions for invertibility of a square matrix to draw conclusions (Don't worry about parts (f) and (i) of Theorem 2.8)

The Assignment Problem and the Hungarian Method (Lecture slides on course web site)

- Convert a case problem to an assignment problem
- Use the Hungarian Method to solve an assignment problem

Game Theory (*ditto*)

- Construct the payoff matrix for a game
- Find the optimal strategies and expected value of a strictly determined game
- Find the optimal strategies and expected value of a 2×2 non-strictly determined game

Linear Programming (problems and links on course web site)

- Convert a case problem to a linear programming problem
- Solve a LP problem with two decision variables using the geometric method

The Simplex Method (links on course web site)

- Convert a linear programming problem to system of linear equations with slack variables
- Solve a linear programming problem with the simplex method